

MATHEMATICAL MODELLING OF TIME-DEPENDENT HEAT FLUX AND TEMPERATURE VARIATION IN A BRAKE DISC DURING BRAKING

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A two-dimensional axisymmetric mathematical model is developed to determine the time-dependent heat flux applied to a brake disc and the resulting temperature variation during braking. The model combines vehicle braking dynamics, the time-dependent decrease in vehicle speed, the relative sliding velocity between the brake disc and pad, and the frictional heat generation rate. The fraction of frictional heat transferred to the disc is estimated using the thermal effusivities of the disc and pad materials. The heat flux acting on the real contact area is converted into an equivalent axisymmetric heat flux while preserving the total thermal power. The temperature field in the brake disc is calculated numerically in MATLAB using the transient heat conduction equation, the initial condition, and convective boundary conditions. In the baseline calculation, the temperature at the selected monitoring point increased from the initial value of 35.0000 °C to 48.3220 °C at 10 s and then decreased to 44.0341 °C at 180 s. The results demonstrate that the developed model can be used to evaluate the relationship between time-dependent thermal loading and temperature variation in the brake disc.

Keywords: brake disc, heat flux, temperature variation, axisymmetric model, sliding velocity, thermal effusivity, heat conduction, MATLAB, braking process.

МОДЕЛСОЗИИ МАТЕМАТИКИИ ЧАРАЁНИ ГАРМИИ ВОБАСТА БА ВАҚТ ВА ТАҒЙИРЁБИИ ҲАРОРАТИ ДИСКИ ТОРМОЗ ҲАНГОМИ ТОРМОЗГИРӢ

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Моделҳои математикӣ дученакаи меҳварсимметрӣ барои муайян намудани ҷараёни гармии вобаста ба вақт, ки ба диски тормоз таъсир мерасонад, ва тағйирёбии ҳарорати он ҳангоми тормозгирӣ таҳия гардид. Модел динамикаи тормозгирии автомобил, коҳишёбии суръати ҳаракат бо гузашти вақт, суръати нисбии лағжиш байни диски тормоз ва колодка, инчунин суръати ҳосилшавии гармӣ дар натиҷаи соишро ба ҳам меафзояд. Ҳиссаи гармии соиш, ки ба диск интиқол меёбад, дар асоси эффузивияти гармии маводи диск ва колодкаи тормоз муайян карда мешавад. Ҷараёни гармие, ки ба масоҳати воқеии тамос таъсир мекунад, бо нигоҳ доштани иқтидори умумии гармӣ ба ҷараёни гармии эквивалентии меҳварсимметрӣ табдил дода мешавад. Майдони ҳарорат дар диски тормоз ба таври ададӣ дар муҳити MATLAB бо истифода аз муодилаи ғайристатсионарии гармигузаронӣ, шартҳои ибтидоӣ ва шартҳои сарҳадии конвективӣ ҳисоб карда мешавад. Дар ҳисоби базавӣ ҳарорат дар нуқтаи назоратии интиҳобшуда аз қимати ибтидоӣ 35,0000 °C то 48,3220 °C дар сонияи 10-ум зиёд гардида, баъдан то 44,0341 °C дар сонияи 180-ум коҳиш ёфт. Натиҷаҳои бадастомада нишон медиҳанд, ки модели таҳияшуда метавонад барои арзёбии робитаи байни сарбории гармии вобаста ба вақт ва тағйирёбии ҳарорати диски тормоз истифода шавад.

Калидвожаҳо: диски тормоз, ҷараёни гармӣ, тағйирёбии ҳарорат, модели меҳварсимметрӣ, суръати лағжиш, эффузивияти гармӣ, гармигузаронӣ, MATLAB, раванди тормозгирӣ.

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НЕСТАЦИОНАРНОГО ТЕПЛОВОГО ПОТОКА И ИЗМЕНЕНИЯ ТЕМПЕРАТУРЫ ТОРМОЗНОГО ДИСКА В ПРОЦЕССЕ ТОРМОЖЕНИЯ

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Разработана двумерная осесимметричная математическая модель для определения изменяющегося во времени теплового потока, действующего на тормозной диск и соответствующего изменения его температуры в процессе торможения. Модель объединяет динамику торможения автомобиля, временное снижение скорости движения, относительную скорость скольжения между тормозным диском и колодкой, а также интенсивность тепловыделения вследствие трения. Доля тепла трения, передаваемого тормозному диску, определяется на основе тепловой эффузивности материалов диска и тормозной колодки. Тепловой поток, действующий на фактическую площадь контакта, преобразуется в эквивалентный осесимметричный тепловой поток при сохранении полной тепловой мощности. Температурное поле тормозного диска рассчитывается численно в среде MATLAB на основе нестационарного уравнения теплопроводности с учётом начального условия и конвективных граничных условий. В базовом расчёте температура в выбранной контрольной точке увеличилась с начального значения 35,0000 °C до 48,3220 °C на 10-й секунде, после чего снизилась до 44,0341 °C к 180-й секунде. Полученные результаты показывают, что разработанная модель может быть использована для оценки взаимосвязи между изменяющейся во времени тепловой нагрузкой и изменением температуры тормозного диска.

Ключевые слова: тормозной диск, тепловой поток, изменение температуры, осесимметричная модель, скорость скольжения, тепловая эффузивность, теплопроводность, MATLAB, процесс торможения.

Introduction

During braking, the kinetic energy of the vehicle is converted into thermal energy due to friction between the brake pad and the disc. The heat generated is distributed between the brake disc, the pad surface and the environment. In this case, the heat flux on the surface of the brake disc does not remain constant during braking, since as the vehicle speed decreases, the relative sliding velocity between the disc and the pad also decreases. Therefore, considering the heat flux on the disc surface as a time-dependent quantity allows a more accurate representation of the thermal processes in the brake mechanism.

During braking, the temperature on the disc surface is not uniformly distributed. The formation of the temperature field is affected by the location of the friction zone, the inner and outer radii of the disc, its thickness, the thermal conductivity, density and specific heat capacity of the material, as well as the heat exchange between the disc and the environment. A local increase in temperature in some parts of the disc changes its thermal state and affects the operating conditions of the brake mechanism. Therefore, it is necessary to use a mathematical model to estimate the time and space distribution of heat in the brake disc.

One of the main issues in calculating the temperature of the brake disc is to correctly determine the heat flux on the disc surface. The heat energy generated during friction depends on the contact pressure, the friction coefficient, and the sliding velocity between the disc and the pad. At the same time, only a certain part of the generated heat is transferred to the disc, while the rest is absorbed by the pad lining. When determining the proportion of heat transferred to the disc, it is necessary to take into account the heat absorption and transfer properties of the disc and lining.

During braking, as the vehicle speed and, accordingly, the sliding velocity decrease, the heat flux also changes over time. After braking, new heat input to the disc surface stops, and the processes of internal heat dissipation in the disc and convective cooling with the environment continue. In such conditions, it is necessary to include the time-dependent heat flux in the boundary condition of the heat transfer equation to determine the temperature change in the disc.

The brake disc is geometrically symmetrical with respect to the axis of rotation. Therefore, expressing its temperature distribution in the radial and through-thickness directions using an axisymmetric model allows simplifying the calculation process. In such a model, the disc is considered as a homogeneous and isotropic body, and the heat generated by friction is given as an external heat flux to the friction surface of the disc. Heat transfer from the disc to the environment is taken into account by means of a convective boundary condition.

When using an axisymmetric model, it is required to distribute the heat flux generated on the real pad-disc contact surface equivalently to the surface of the friction ring of the disc. In this case, the equality of the total heat power in the real contact surface and the axisymmetric calculation area must be maintained. This approach allows the heat flux in the real contact area to be included in the two-dimensional radial-thickness calculation model.

The purpose of this study is to determine the time-dependent heat flux on the surface of the brake disc and develop a mathematical model of the temperature change in the disc based on the relationship between braking dynamics, the sliding velocity between the disc and the pad, the heat generated by friction, and the fraction of heat transferred to the disc.

To achieve this objective, the vehicle-speed history and disc-pad sliding velocity are determined, the frictional heat flux and the fraction transferred to the disc are evaluated, the real contact heat flux is transformed into an equivalent axisymmetric load, and the transient heat-conduction problem is solved numerically in MATLAB.

As a result of this study, a single mathematical calculation sequence is formed that relates the braking conditions to the heat flux on the disc surface and the time-dependent temperature field of the disc. The main result of the model is to determine the temperature distribution of the brake disc in the radial and thickness directions over time.

During braking, the kinetic energy of the vehicle is converted into thermal energy due to friction between the disc and the pad. The overall energy balance of the process is expressed as:

$$E_k = \frac{mv_0^2}{2} \quad (1)$$

Here, m — calculated mass of the vehicle, kg; v_0 — speed at the beginning of braking, m/s. Some of the energy produced is absorbed by the brake disc and pad, while the rest is transferred to the environment through convection, radiation, and thermal conduction.

Belhocine and Bouchetara [1] developed a three-dimensional model of solid and ventilated automotive brake discs in ANSYS and showed that the brake mode, geometric structure, and thermophysical properties of the material affect the disc temperature. This study substantiates the need to consider not only the total energy dissipated, but also the disc geometry and cooling conditions when evaluating the temperature state of the brake disc.

Previous studies have employed 1D analytical, 2D axisymmetric, and 3D non-axisymmetric models to investigate thermal processes in brake discs. One-dimensional models allow for a simple and quick assessment of the effect of time-dependent friction loading on temperature, but do not determine the temperature distribution in the radial and circumferential directions of the disc. Three-dimensional models represent the local contact zone and maximum temperature in detail, but require large computational resources. The two-dimensional axisymmetric approach allows for the determination of the temperature field in the radial and thickness directions of the disc with relatively low computational complexity. Therefore, the 2D axisymmetric model was chosen in this study.

Yevtushenko, Kuciej and Topczewska [2] analyzed ten different time profiles of friction force analytically. The results showed that even under the same total energy conditions, the shape of the $q(t)$ function is affected by the surface temperature, the maximum temperature and the temperature gradient along the thickness. Therefore, taking the heat flux on the brake disc as a constant value may not accurately describe the temperature process.

Topczewska, Gerlici and Yevtushenko [3] proposed an analytical model that takes into account the initial increase in contact pressure, the average friction coefficient and the time-dependent speed during a single braking. The model is based on a one-dimensional semi-infinite body problem and is verified with thermocouple results obtained on a full-scale dynamometer stand. This approach confirms that contact pressure, friction coefficient, and time-dependent sliding velocity are key factors governing the thermal load.

Not all frictional heat is transferred to the brake disc. Under ideal thermal contact, the heat partition between the disc and pad can be estimated from their thermal effusivities. Under real contact conditions, however, surface roughness, thermal contact resistance, actual contact area, and pressure non-uniformity may also affect the heat partition [3, 4].

Table 1 – Physical factors considered in thermal models

Source	$q(t)$	Heat distribution in the disc-pad	Convection	Radiation	Experimental investigation
[1]	✓	✓	✓	–	–
[2]	✓	✓	–	–	–
[3]	✓	✓	–	–	✓
[4]	✓	✓	✓	–	–
[5]	✓	–	✓	–	–
[6]	✓	partially	✓	–	✓
[7]	✓	partially	✓	✓	✓
[8]	✓	✓	✓	–	Numerical comparison
Proposed model	✓	✓	✓	–	Next step

Table 1 shows that time-dependent heat flux and convective cooling are considered as the main physical factors in most of the models considered. The heat distribution between the disc and the pad is also considered in most studies, since not all the heat generated during friction is transferred to the disc. Radiative heat transfer is included only in some models, mainly those focused on high temperatures or long-term braking. Since this study considered a single and relatively short braking, the main attention was

paid to the time-dependent friction load, heat distribution between the disc and the pad, and convective cooling. Radiation was not considered in order to simplify the model. Experimental verification is planned for a later stage of the study.

Gao and Lin [4] investigated the non-axisymmetric three-dimensional temperature field of a brake disc by representing the moving pad as a local heat source. Their results showed that the disc temperature varies significantly in the circumferential direction and that local maximum temperatures develop near the friction source, indicating that a 2D axisymmetric model cannot fully reproduce local hot spots. In contrast, Wolff and Kukulski [6] applied a two-dimensional axisymmetric transient thermal model using the finite element method and validated the results against full-scale dynamometer data, demonstrating that this approach can effectively determine temperature variations in the radial and thickness directions of the disc with considerably lower computational cost. Wu et al. [7] also employed a symmetric two-dimensional formulation based on the barycentric rational interpolation collocation method and additionally considered radiative heat transfer together with convection. Overall, 2D axisymmetric models provide a practical balance between computational efficiency and the ability to describe the principal radial and through-thickness temperature variations in a brake disc. However, because circumferential variations are neglected, such models are inherently limited in predicting moving local heat sources, non-uniform circumferential temperature fields, and local hot spots that can be captured more accurately by three-dimensional non-axisymmetric models.

In local and regional studies, Inagamov et al. [8] investigated temperature and thermal stresses in a railway wheel–shoe braking system using COMSOL, demonstrating the importance of contact pressure, velocity, and material properties in thermal calculations. Tilavov [9] discussed the conversion of kinetic energy into heat in automotive braking systems, although a time-dependent spatial thermal model was not developed.

Previous studies have demonstrated the effectiveness of analytical, two-dimensional axisymmetric, and three-dimensional non-axisymmetric approaches for analysing thermal processes in braking systems. However, the principal components of the braking process, including vehicle deceleration, time-dependent sliding velocity, frictional heat generation, heat partition between the disc and pad, and transient heat conduction in the disc, are often considered separately or under simplified thermal loading conditions. Furthermore, although three-dimensional models can reproduce circumferential temperature non-uniformity and local hot spots, they require substantially greater computational resources.

In the present study, these physical processes are integrated within a unified two-dimensional axisymmetric mathematical model. The model links vehicle braking dynamics with the time-dependent disc–pad sliding velocity and frictional heat generation, accounts for heat partition between the disc and pad on the basis of their thermal effusivities, and converts the heat flux acting over the actual contact area into an equivalent axisymmetric heat flux while preserving the total thermal power. The resulting transient temperature field is then determined numerically from the heat conduction equation with convective boundary conditions. This approach provides a computationally efficient framework for evaluating the relationship between time-dependent thermal loading and the radial and through-thickness temperature variation of the brake disc, while acknowledging that local circumferential hot spots are beyond the scope of the axisymmetric formulation.

Materials and methods

Physical scheme of the mathematical model and accepted assumptions

During braking, the kinetic energy of the vehicle is converted into thermal energy due to the friction between the brake disc and the pad. Part of the heat generated is transferred to the disc, part to the pad, and the rest is transferred to the environment. The heat flux on the disc surface changes over time as the speed decreases during braking.

The physical sequence of the model is expressed as follows:

$$E_k \rightarrow P_f(t) \rightarrow q_{\text{real}}(t) \rightarrow q_{\text{axi}}(t) \rightarrow T(r, z, t) \quad (2)$$

Here:

- E_k — initial kinetic energy of the car, J;
- $P_f(t)$ — Power of the friction, W;

- $q_{\text{real}}(t)$ — heat flux on the real disc-pad contact surface, W/m²;
- $q_{\text{axi}}(t)$ — equivalent heat flux to be included in the axisymmetric model, W/m²;
- $T(r, z, t)$ — Time-dependent temperature field on the brake disc, °C.

Computational domain

The brake disc is considered as a hollow cylinder with an inner radius R_i , an outer radius R_o and a total thickness of $2b$. The calculation area:

$$R_i \leq r \leq R_o, \quad -b \leq z \leq b, \quad t \geq 0 \quad (3)$$

is determined in the intervals.

Here:

- r — radial coordinate, m;
- z — coordinate by disc thickness, m;
- b — half-thickness of the disc, m;
- t — time, s.

The heat flux applied to the disc surface is distributed over time in the radial and thickness directions.

In the axisymmetric approach, the temperature change along the circular coordinate is not taken into account:

$$\frac{\partial T}{\partial \theta} = 0 \quad (4)$$

Therefore, the general three-dimensional:

$$T = T(r, \theta, z, t) \quad (5)$$

The temperature field is represented in the following two-dimensional form:

$$T = T(r, z, t) \quad (6)$$

This simplification serves to determine the equivalent temperature distribution along the friction ring of the disc. The model calculates the temperature field in the radial and through-thickness directions, rather than local circumferential hot spots.

Assumptions Adopted in the Model

The main assumptions adopted in formulating the model are presented in Table 2.

Table 2 – Key assumptions made in the mathematical model

№	Assumption	Mathematical expression or content	Impact on the model
1	The disc is homogeneous and isotropic	$k_d, \rho_d, c_d = \text{const}$	Properties are assumed to be the same in all directions
2	There is no volumetric heat source inside the disc	$\dot{q}_v = 0$	Heat is introduced only through the surface
3	The model is axisymmetric	$\partial T / \partial \theta = 0$	Unevenness in the circumferential direction is not taken into account
4	The contact pressure is represented by an average value	$p(t) \approx p_{\text{eq}}$	Real pressure fluctuations are averaged
5	The speed decreases linearly during braking	$v(t) = v_0 - at$	Heat flux decreases with time
6	Heat is partitioned between the disc and the brake-pad lining	$q_d(t) = \eta_d q_f(t)$	Only a part of the generated heat is transferred to the disc
7	The actual contact heat flux is distributed equivalently over the friction annulus	$q_{\text{real}} A_c = q_{\text{axi}} A_{\text{ann}}$	Total thermal power is conserved
8	The cooling is represented by convection	$q_{\text{conv}} = h(T - T_{\infty})$	Heat loss from the disc to the environment is taken into account
9	No new heat is introduced after braking is over	$q(t) = 0, t > t_b$	In the next stage, only diffusion and cooling continue

The assumption that the disc is considered a homogeneous and isotropic body, that there is no internal volumetric heat source, and that frictional heat is given in the form of an external surface load is consistent with the initial model assumptions.

Energy transfer in the brake disc

The initial kinetic energy of the car during braking is:

$$E_k = \frac{mv_0^2}{2} \quad (7)$$

Here, m — effective vehicle mass, kg; v_0 — speed at the beginning of braking, m/s.

This energy is distributed among all the braking mechanisms of the car, the resistive forces, and the disc and the pad. Therefore, the value of E_k cannot be directly equated to the thermal energy entering a single brake disc. In this model, the heat transferred to the disc is determined not from the total kinetic energy of the car, but from the friction force at the disc-pad contact.

The total energy balance of the brake disc is:

$$\frac{dU_d}{dt} = \dot{Q}_{in}(t) - \dot{Q}_{conv}(t) \quad (8)$$

written as above. Here, U_d — internal energy stored in the disc; $\dot{Q}_{in}$ — heat energy entering the disc through friction; $\dot{Q}_{conv}$ — the heat energy convectively transferred from the disc surface to the surroundings.

Thermal conductivity within the disc is not an energy loss, but a redistribution of heat in the radial and thickness directions. Therefore, it is not separately isolated in the overall energy balance of the disc, but is taken into account in the heat conduction equation.

Thus, the heat transferred to the disc is determined not directly from the total kinetic energy of the car, but through the friction force at the disc-pad contact.

Braking dynamics and sliding velocity determination

The heat flux on the disc surface depends on the tangential sliding velocity of the disc relative to the pad. Therefore, the average deceleration of the vehicle, the duration of braking, and the change in speed over time are initially determined. In this model, the calculation was organized in the sequence of first determining the braking distance and average deceleration, then the vehicle speed and the sliding velocity between the disc and the pad.

Average deceleration and braking duration

Let the car travel the braking distance from the initial speed v_0 until it comes to a complete stop. Here, the braking distance is the distance traveled from the moment the brake mechanism is activated until the car stops, not the driver reaction distance.

On a flat road and assuming a constant average deceleration:

$$v^2 = v_0^2 - 2aS \quad (9)$$

Since $v = 0$ at the end of braking:

$$a = \frac{v_0^2}{2S} \quad (10)$$

Here, a — average deceleration of the car, m/s².

Braking duration:

$$t_b = \frac{v_0}{a} \quad (11)$$

or considering expression (10):

$$t_b = \frac{2S}{v_0} \quad (12)$$

Change in car speed over time

It is assumed that the vehicle speed decreases linearly during braking:

$$v(t) = \begin{cases} v_0 - at, & 0 \leq t \leq t_b, \\ 0, & t > t_b. \end{cases} \quad (13)$$

This simplification represents the time-varying average of the real deceleration. A linear decrease in speed is also assumed in this mathematical model.

Angular velocity of the disc

The model does not lock the wheel during braking and does not take into account the longitudinal sliding of the tire relative to the road. In such conditions:

$$v(t) = \omega_w(t)R_w \quad (14)$$

Here, $\omega_w(t)$ — angular velocity of the wheel, rad/s; R_w — effective radius of the wheel, m. Since the brake disc is kinematically connected to the wheel hub:

$$\omega_d(t) = \omega_w(t) = \frac{v(t)}{R_w} \quad (15)$$

From this:

$$\omega_d(t) = \begin{cases} \frac{v_0 - at}{R_w}, & 0 \leq t \leq t_b, \\ 0, & t > t_b. \end{cases} \quad (16)$$

If a wheel locks or there is significant tire slip, the relationship $v = \omega_w R_w$ will be broken. This situation is not included in the scope of this simplified model.

Average friction radius

The tangential velocity of the disc surface depends on the radial coordinate:

$$v_r(r, t) = \omega_d(t)r \quad (17)$$

The average friction radius R_m is used to express the velocity in the contact zone with a single value. When the contact pressure is assumed to be uniformly distributed in the radial direction:

$$R_m = \frac{2 R_{f,o}^3 - R_{f,i}^3}{3 R_{f,o}^2 - R_{f,i}^2} \quad (18)$$

Here $R_{f,i}$ and $R_{f,o}$ — inner and outer radii of the friction ring.

If R_m is previously determined based on the actual geometry of the brake disc and pad, this value is used in the numerical model.

Disc-pad sliding velocity

When the pad is considered stationary and the disc is in rotational motion, the tangential sliding velocity of the disc friction surface relative to the pad is:

$$v_s(t) = \omega_d(t)R_m \quad (19)$$

Considering expression (17):

$$v_s(t) = \frac{R_m}{R_w} v(t) \quad (20)$$

and during braking:

$$v_s(t) = \frac{R_m}{R_w} (v_0 - at), \quad 0 \leq t \leq t_b \quad (21)$$

The complete time-dependent function is:

$$v_s(t) = \begin{cases} \frac{R_m}{R_w} (v_0 - at), & 0 \leq t \leq t_b, \\ 0, & t > t_b. \end{cases} \quad (22)$$

At the beginning of braking:

$$v_s(0) = \frac{R_m}{R_w} v_0 \quad (23)$$

and at the end of braking:

$$v_s(t_b) = 0 \quad (24)$$

Determining the average friction radius on the disc and the sliding velocity through the wheel radius is one of the key steps in the model.

Determining time-dependent heat flux

The friction force generated at the disc-pad contact depends on the contact pressure, the coefficient of friction, and the sliding velocity. The normal force acting on the pad and the friction force are:

$$N = p_{eq} A_c, P_f(t) = \mu p_{eq} A_c v_s(t) \quad (25)$$

Here, p_{eq} — average equivalent contact pressure; A_c — disc–pad contact surface; μ — coefficient of friction.

Only η_d part of the heat generated by friction is transferred to the disc. Therefore, the heat flux on the real contact surface is:

$$q_{real}(t) = \eta_d \mu p_{eq} v_s(t) \quad (26)$$

Substituting the sliding velocity obtained in the previous section gives:

$$q_{real}(t) = \begin{cases} \eta_d \mu p_{eq} \frac{R_m}{R_w} (v_0 - at), & 0 \leq t \leq t_b, \\ 0, & t > t_b \end{cases} \quad (27)$$

is obtained. Therefore, as the speed decreases during braking, the heat flux also decreases linearly, and when braking is over, no new frictional heat enters the disc. In this study, the heat flux is determined by the fraction of heat transferred to the disc, the contact pressure, the coefficient of friction, and the time-dependent sliding velocity.

When the disc and the pad are considered as semi-infinite bodies in ideal thermal contact, the fraction of heat transferred to the disc is estimated by the thermal effusivity of the materials:

$$e = \sqrt{k\rho c}, \eta_d = \frac{e_d}{e_d + e_p} \quad (28)$$

Here, k , ρ and c — thermal conductivity of the material, density and specific heat capacity; e_d and e_p — thermal effusivity of the disc and pad. This assessment is based on the assumption of ideal contact and does not directly take into account the thermal resistance of contact and surface roughness in real conditions.

In a real brake system, the pad is in contact with only a limited part of the disc circumference. In an axisymmetric model, the heat flux is distributed over the entire friction ring. In this study, A_c denotes the total nominal contact area of the pads on both sides of the disc, and A_{ann} denotes the total surface area of the friction rings on both sides of the disc. If the nominal contact area of one pad is A_p , then for a double-sided disc brake, $A_c = 2A_p$ is taken. From the condition of conservation of total heat energy:

$$q_{real}(t) A_c = q_{axi}(t) A_{ann} \quad (29)$$

$$q_{axi}(t) = \frac{A_c}{A_{ann}} q_{real}(t) \quad (30)$$

where is the total area of the two friction surfaces:

$$A_{ann} = 2\pi(R_{f,o}^2 - R_{f,i}^2) \quad (31)$$

As a result, the heat flux introduced into the axisymmetric model is:

$$q_{\text{axi}}(t) = \begin{cases} \frac{A_c}{A_{\text{ann}}} \eta_d \mu p_{\text{eq}} \frac{R_m}{R_w} (v_0 - at), & 0 \leq t \leq t_b, \\ 0, & t > t_b \end{cases} \quad (32)$$

This equivalence adapts the loading to the axisymmetric calculation domain while conserving the total thermal power over the actual contact area.

The determined function $q_{\text{axi}}(t)$ is introduced into the boundary condition of the heat transfer equation in the brake disc in the next step.

Mathematical model of heat conduction in the brake disc

In the axisymmetric approach, the disc temperature depends on the radial coordinate, the thickness coordinate, and time:

$$T = T(r, z, t) \quad (33)$$

Assuming that the disc is homogeneous and isotropic, and that there is no volumetric heat source inside it, the unsteady heat transfer equation is expressed as follows:

$$\rho_d c_d \frac{\partial T}{\partial t} = k_d \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] \quad (34)$$

where k_d , ρ_d and c_d are the thermal conductivity, density, and specific heat capacity of the disc material, respectively. This equation describes the temperature change in the radial and thickness directions of the disc.

Computational domain:

$$R_i \leq r \leq R_o, \quad -b \leq z \leq b \quad (35)$$

is taken in the range. Here R_i and R_o are the inner and outer radius of the disc, and $2b$ is its total thickness.

The assumption of the disc as a hollow cylinder is consistent with the initial model.

The heat transfer on the disc surface is expressed by a single general boundary condition:

$$k_d \frac{\partial T}{\partial n} = q_s(r, t) - h(T - T_\infty) \quad (36)$$

here n — external normal direction of the surface; h — convective heat transfer coefficient; T_∞ — ambient temperature.

Surface heat flux:

$$q_s(r, t) = \begin{cases} q_{\text{axi}}(t), & R_{f,i} \leq r \leq R_{f,o}, \\ 0, & \text{"outside the friction zone"}. \end{cases} \quad (37)$$

Thus, in the friction ring, heat input to the disc and convective cooling are taken into account together. On the remaining surfaces, only convective heat exchange with the environment occurs. In the model, the time-dependent heat flux and convective cooling are given by a single boundary condition.

Initial condition:

$$T(r, z, 0) = T_0 \quad (38)$$

is taken as, where T_0 is the disc temperature before the start of braking.

After braking is complete:

$$q_{\text{axi}}(t) = 0, \quad t > t_b \quad (39)$$

and only internal heat dissipation and convective cooling continue in the disc.

As a result, the mathematical problem consists of the following system:

$$\boxed{\begin{aligned} \rho_d c_d \frac{\partial T}{\partial t} &= k_d \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right], \\ k_d \frac{\partial T}{\partial n} &= q_s(r, t) - h(T - T_\infty), \\ T(r, z, 0) &= T_0. \end{aligned}} \quad (40)$$

This system is introduced with $q_{\text{axi}}(t)$, which was determined in the section on determining the time-dependent heat flux, and the disc:

$$T(r, z, t)$$

is the temperature field. This equation was solved numerically in MATLAB.

The formulated mathematical problem is solved numerically in the MATLAB program in the next section.

Numerical solution algorithm in MATLAB

The heat transfer equation is solved numerically in MATLAB. In this case, a two-dimensional, axisymmetric model of the disc section is assumed and discretized in the radial and thickness directions:

$$r_i = R_i + (i - 1)\Delta r, \quad z_j = -b + (j - 1)\Delta z \quad (41)$$

$$\Delta r = \frac{R_o - R_i}{N_r - 1}, \quad \Delta z = \frac{2b}{N_z - 1} \quad (42)$$

here N_r and N_z — number of nodes by radius and thickness.

According to the initial condition, at the initial time of the calculation, the temperature is assumed to be uniform throughout the entire cross section of the disc and equal to:

$$T_{i,j}^0 = T_0 \quad (43)$$

At each time step, the vehicle speed and the sliding velocity between the disc and pad are first determined, after which the time-dependent axisymmetric heat flux $q_{\text{axi}}(t)$ is calculated. This heat flux is applied only to the surface nodes located within the disc–pad contact region. Once braking is completed, no additional frictional heat is introduced, and only internal heat conduction and convective cooling are considered.

The axisymmetric heat flux is applied only to the surface nodes located within the friction annulus:

$$R_{f,i} \leq r \leq R_{f,o}$$

When the braking is over, i.e. when $t > t_b$, the new heat input is stopped and only the heat dissipation and convective cooling inside the disc are continued in the calculation. The calculation in the model is also organized in the same order.

During the calculation, the velocity, sliding velocity and heat flux are updated at each time step, and then the temperature values at the disc nodes are calculated. As a result, the time-dependent temperature field along the disc and the temperature change at the selected control point are obtained.

Thus, the numerical solution establishes the following computational relationship:

$$q_{\text{axi}}(t) \rightarrow T(r, z, t)$$

Baseline computational conditions

To demonstrate the application of the developed mathematical model, a single baseline case corresponding to a metal-based brake lining was adopted. This calculation was not performed to compare materials, but to demonstrate the time-dependent thermal load and temperature change in the brake disc.

Table 3 – Main parameters used in the baseline calculation

Parameter	Symbol	Value
Initial speed	v_0	16.66 m/s
Ratio of average friction radius to wheel radius	R_m/R_w	0.32334
Initial sliding velocity	$v_s(0)$	5.3869 m/s
Fraction of heat transferred to the disc	η_d	0.8226
Combined area of the two friction annuli	A_{ann}	0.05485 m ²
Surface equivalence coefficient	k_A	0.07384
Initial disc temperature	T_0	35 °C
Calculation duration	t_{max}	180 s

The principal parameters adopted for the baseline case are listed in Table 3: $\eta_d = 0.8226$, $R_m/R_w = 0.32334$, $v_s(0) = 5.3869$ m/s, $A_{\text{ann}} = 0.05485$ m² and $k_A = 0.07384$.

Thermophysical properties of metal-based brake-pad lining:

$$k_p = 5 \text{ W/(m} \cdot \text{K)}, \quad \rho_p = 1400 \text{ kg/m}^3, \quad c_p = 1000 \text{ J/(kg} \cdot \text{K)}$$

These values are used to determine the fraction of heat transferred to the disc based on thermal effusivity.

Based on the adopted parameters, the initial heat flux on the real contact surface is:

$$q_{\text{real},0} = 5.5834 \times 10^6 \text{ W/m}^2$$

The initial equivalent heat flux introduced into the axisymmetric model is:

$$q_{\text{axi},0} = 4.1230 \times 10^5 \text{ W/m}^2$$

The calculation was performed for 180 s. The heat flux was applied only to the friction zone of the disc during braking, after which heat dissipation and convective cooling within the disc were taken into account. The temperature change over time was recorded at the monitoring point P4 on the disc surface.

Results

Based on the adopted baseline parameters and the time-dependent heat flux, the temperature change in the brake disc was calculated. During braking, the heat flux decreased with speed and became zero at $t = t_b$. After that, no new heat was introduced into the disc, and the temperature change was determined by heat dissipation and convective cooling within the disc.

Table 4 – Temperature change over time at monitoring point P4

Time, s	Temperature, °C
0	35.0000
10	48.3220
20	47.5079
30	47.0000
60	45.8872
90	45.1304
120	44.6201
180	44.0341

The change in temperature over time at monitoring point P4 is graphically presented in Figure 1.

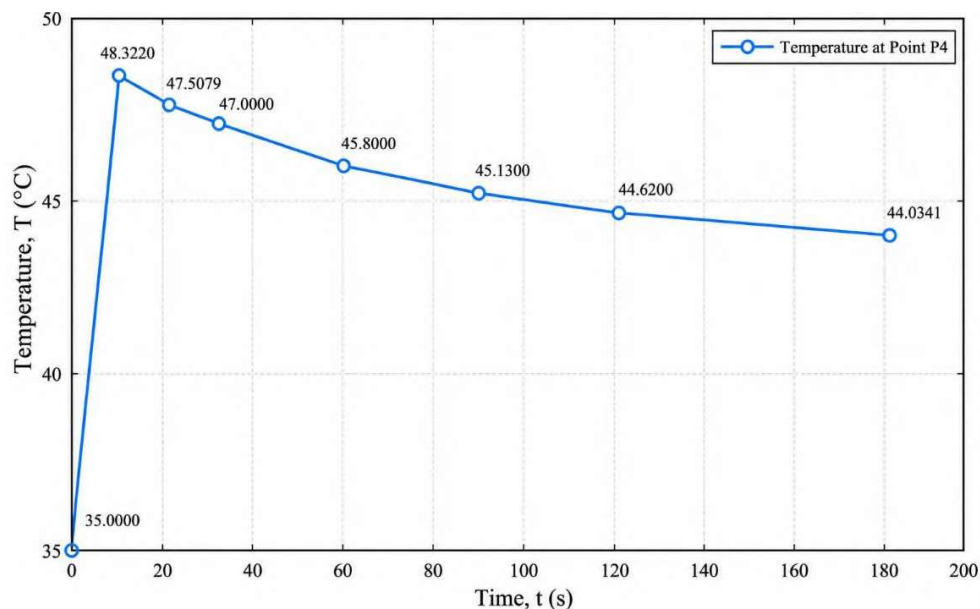


Figure 1 – Temperature variation over time at monitoring point P4 for a metal-based brake lining

Figure 1 shows that the temperature at monitoring point P4 increased from 35.0000 °C to 48.3220 °C in the first 0–10 seconds. At subsequent selected time points, the temperature was 47.5079 °C at 20 seconds, 47.0000 °C at 30 seconds, and then decreased steadily. At 180 seconds, the temperature at the control point was 44.0341 °C.

The highest calculated temperature at the selected time points was observed at 10 seconds. Temperature increase compared to the initial temperature:

$$\Delta T_{10} = T_{10} - T_0 = 48.322 - 35 = 13.322 \text{ }^{\circ}\text{C}$$

Temperature drop between 10–180 seconds:

$$\Delta T_{10-180} = T_{10} - T_{180} = 48.3220 - 44.0341 = 4.2879 \text{ }^{\circ}\text{C}$$

At the end of the calculation, the difference between the temperature at 180 seconds and the initial temperature is:

$$\Delta T_{0-180} = T_{180} - T_0 = 44.0341 - 35.000 = 9.0341 \text{ }^{\circ}\text{C}$$

Thus, at the end of the calculation, it was determined that the disc temperature increased by 9.0341 °C compared to the initial state.

The results obtained show that time-dependent thermal loading leads to an increase in the disc temperature, and after the loading is completed, the temperature decreases due to heat dissipation and convective heat exchange within the disc.

Discussions

The baseline calculation results showed that the heat flux on the disc varies depending on the vehicle speed. In the adopted linear deceleration model, the sliding velocity and the associated axisymmetric heat flux decrease during braking and become zero at the end of braking. This result is consistent with the formulated mathematical relationships.

The temperature at monitoring point P4 increased from 35.0000 °C to 48.3220 °C in the first 0–10 seconds. In the subsequent time interval, the temperature decreased and was 47.5079 °C at 20 seconds, 47.0000 °C at 30 seconds, and 44.0341 °C at 180 seconds. Thus, at the selected time points, an initial heating and then a cooling process were observed. At the end of the calculation, at 180 seconds, the disc temperature was 9.0341 °C higher than the initial value.

The obtained values are related to the selected baseline parameters, the assumed thermophysical properties and the model assumptions. The two-dimensional axisymmetric model takes into account the radial and thickness heat dissipation in the disc, but does not reflect the local temperature irregularities in the circumferential direction. This is due to the fact that the heat flux on the real contact surface is distributed equivalently over the entire friction ring.

The contact pressure, friction coefficient and thermophysical properties of the materials are taken as constant effective parameters in the model. The contact thermal resistance, surface roughness and temperature-dependent changes in the parameters are not taken into account separately. Therefore, the model does not serve to fully represent all local processes in a real brake mechanism, but to evaluate the main connection between the time-dependent heat flux and the temperature change in the disc.

Thus, in the basic calculation, the temperature change corresponding to the assumed physical connections and boundary conditions of the model was obtained. The developed model can serve as a theoretical basis for comparison with further experimental results and for assessing the influence of key parameters on disc temperature.

Conclusion

In this study, a two-dimensional axisymmetric mathematical model was developed to determine the time-dependent heat flux acting on the brake disc and the temperature field on the disc during braking.

The model determined the vehicle deceleration, the time-dependent speed change, and the sliding velocity between the disc and the pad based on the braking distance. The heat flux generated by friction was expressed in terms of the heat transfer fraction to the disc, the contact pressure, and the friction coefficient. The heat flux on the real contact surface was adapted to the axisymmetric friction ring while conserving the total thermal power.

The temperature field on the disc was calculated numerically in MATLAB based on the transient heat-conduction equation, initial conditions, and convective boundary conditions. In the baseline calculation, the initial temperature at the monitoring point P4 was 35.0000 °C and increased to 48.3220 °C at the 10th second. The temperature decreased in the next time interval, reaching 44.0341 °C at 180 seconds. The temperature decrease in the period from 10 to 180 seconds was 4.2879 °C, and at the end of the calculation, the temperature was found to be 9.0341 °C higher than the initial value. The results

showed that it is possible to estimate the relationship between the time-dependent thermal load and the temperature change in the brake disc under the conditions of the adopted model. The developed model can serve as a basis for theoretical studies of the influence of the main parameters on the temperature field in the disc and for comparison with subsequent experimental results.

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